CS 7800: Advanced Algorithms

Class 1: Intoduction + Stable Matching

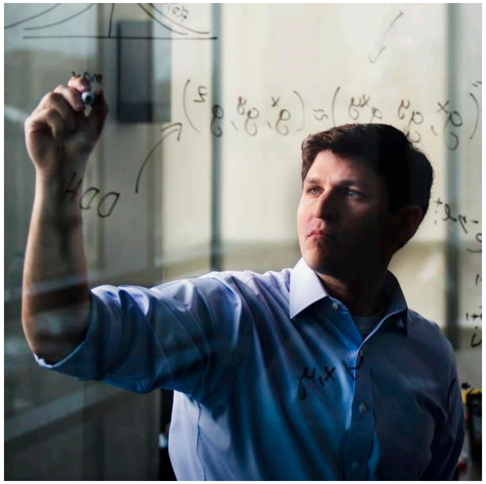
Jonathan Ullman
September 5, 2025

Me

Jonathan Ullman

- Feel free to call me Jon
- Research: Foundations of Trustworthy AI and Statistics

- Office: 177 Huntington 616
- Office Hours:
  - Tricky because I'm in 177
  - Will poll for a good time
  - Always available by appt



The TA Team

John Abascal

- Will help us part-time
- He has an adorable sausage dog

- Office: 177 Huntington 6th Floor

Introductions!

Algorithms

What is an algorithm?

> *An explicit, precise, unambiguous, mechanically-executable sequence of elementary instructions for solving a computational problem. -Jeff Erickson*

Algorithms

What is algorithms (the subfield of CS)?

> *The rigorous mathematical study of computational problems and the algorithms for solving them.*

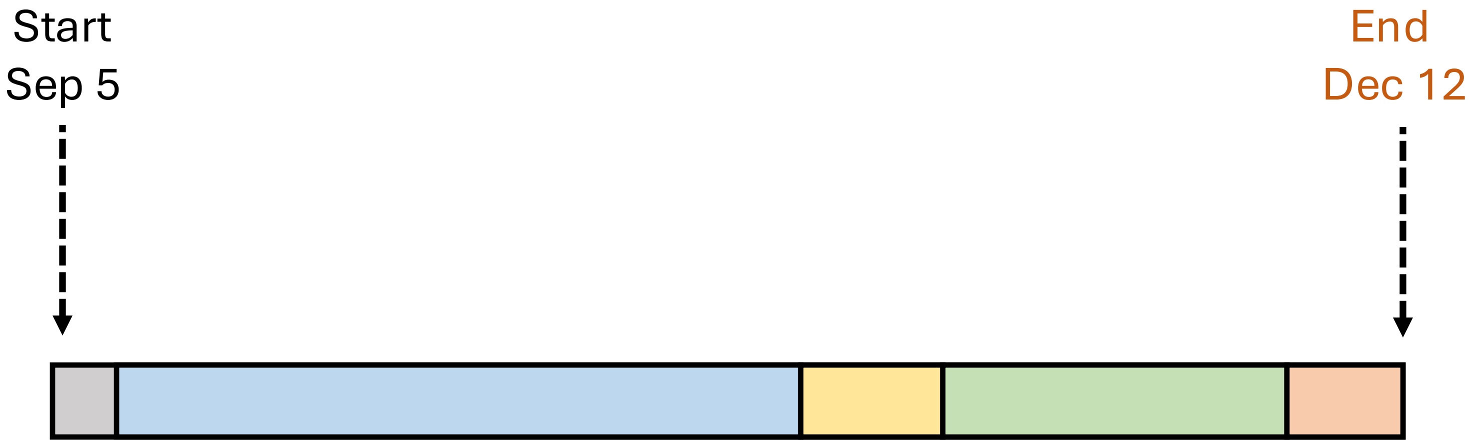
Algorithms

What is CS 7800: Advanced Algorithms?

(1) An overview of the most fundamental algorithms and techniques that we believe every PhD computer scientist should know.

(2) A mental workout to help you develop analytic and mathematical reasoning and communication skills for computer science research.

Course Structure



Start
Sep 5

End
Dec 12

Stable Matching

Optimization

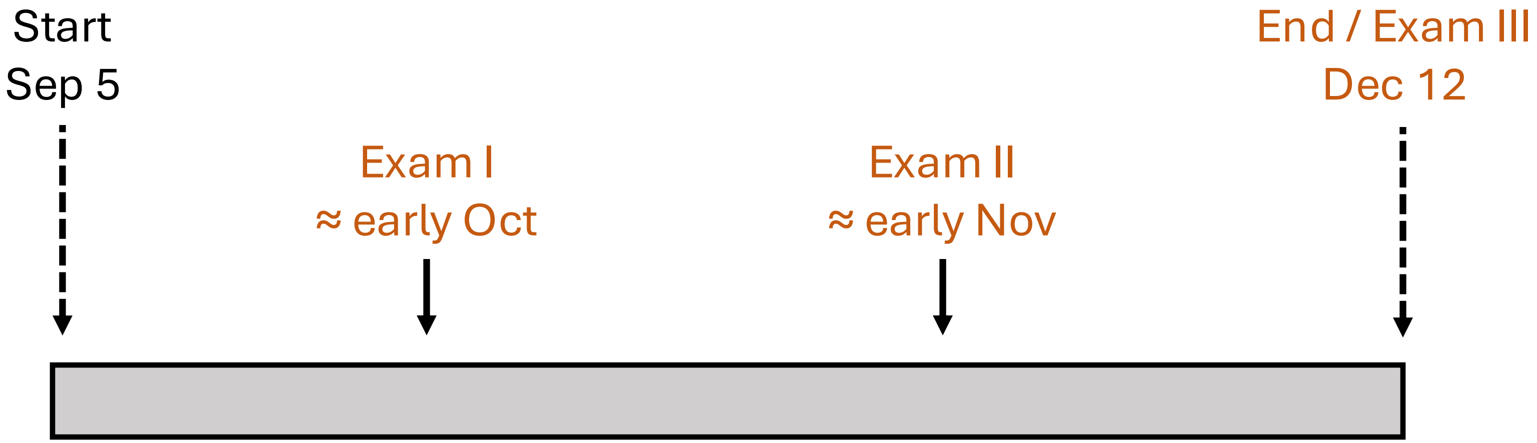
- Greedy
- Dynamic programming
- Network flow
- Linear programming
- Convex optimization

Intractability

Randomized

Misc

Course Structure



Start
Sep 5

Exam I
≈ early Oct

Exam II
≈ early Nov

End / Exam III
Dec 12

Evaluation:

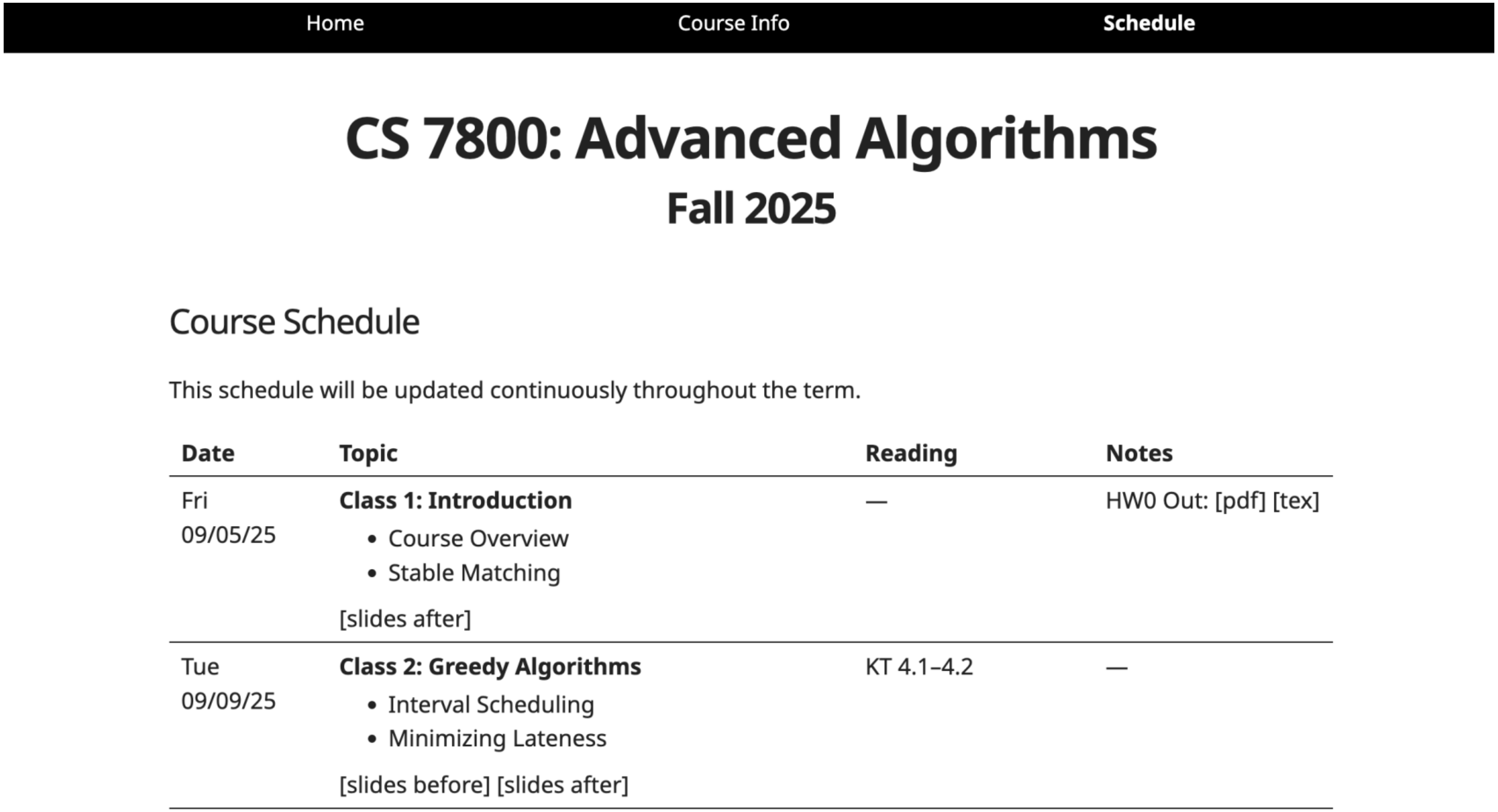
- 3x exams = 75%
  - Not cumulative but the material builds on itself
- 6x assignments = 25%
  - Drop the lowest score

Grading:

- Standard scale (e.g. A/A- is 90%+)
- Generously curved as needed
- Typical distribution:
  - 50% get A/A-, 50% get B+/B
  - I’m more generous with small classes

Course Website

http://jonathan-ullman.github.io/cs7800-f25



CS 7800: Advanced Algorithms

Fall 2025

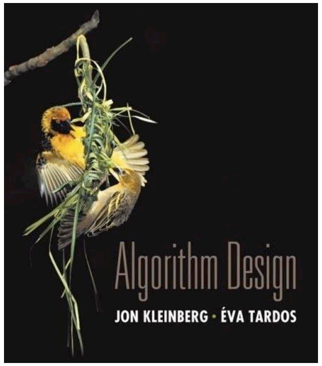
Course Schedule

This schedule will be updated continuously throughout the term.

Date	Topic	Reading	Notes
Fri 09/05/25	**Class 1: Introduction** • Course Overview • Stable Matching [slides after]	—	HW0 Out: [pdf] [tex]
Tue 09/09/25	**Class 2: Greedy Algorithms** • Interval Scheduling • Minimizing Lateness [slides before] [slides after]	KT 4.1–4.2	—

Recommended Resources

- Algorithm Design by Kleinberg and Tardos
  - We’ll follow this closely in the 1st half
  - Can easily find copies



- Algorithms by Jeff Erickson
  - Useful for review, alternate perspective, and some advanced topics
  - Will use this more in the 2nd half
  - Free on the web

Assignments

- 6 HW Assignments (probably)
  - Approximately every two weeks
  - Late days: total of 5, max of 2 per assignment
  - Further extensions granted for *special* circumstances

- All questions are algorithms and related mathematics, no programming

- **Review ~~HW0~~ HW1 out now, due Friday 9/12 at 11:59pm!**
  - No late days—I want to quickly test your background

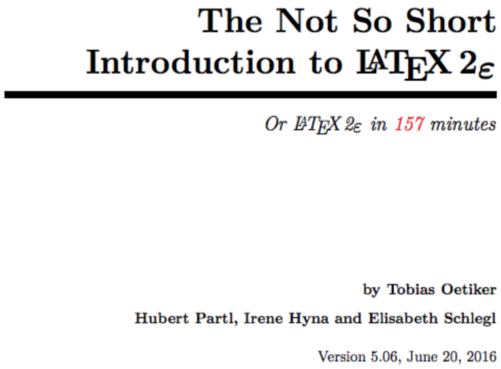
Assignment Philosophy/Policies

- This course has two related-yet-different goals
    - #1: give a working knowledge of algorithms (everyone has to)
    - #2: exercise and stretch your brain (you get out what you put in)
- Exams are for #1 and are most of the evaluation
- Homework is to prepare you for exams and for #2
    - A few *assigned/graded* problems so you get feedback
    - More *optional/ungraded* problems so you can get exercise

- AI/Honestly Policy: *You're adults and scholars, act like it*
    - You can easily ace the assignments using AI, I can't reliably stop you
    - Using AI won't prepare you for exams, which are most of your grade
    - Using AI won't make you a better scholar
    - Using AI wastes my time giving feedback

Assignment Logistics

- Homework must be typeset in LaTeX!
  - You’ll have to learn it sometime!
  - Many good resources available
  - Many good editors available (Overleaf, TexStudio)
  - I will provide source to get you started



The Not So Short Introduction to LaTeX 2_ε

Or LaTeX 2_ε in 157 minutes

by Tobias Oetiker
Hubert Partl, Irene Hyna and Elisabeth Schlegl

Version 5.06, June 20, 2016

Assignment Logistics

- I use Gradescope for homework
  - Entry code: **D3ERDX**

Discussion Forum

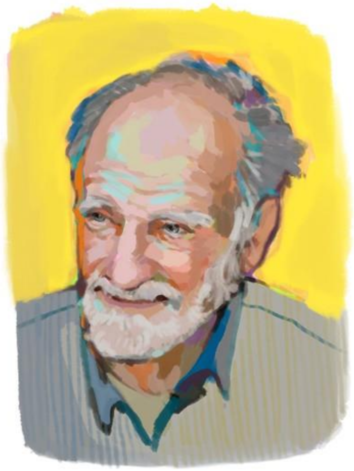
- I’ve used Piazza in the past but I’m open minded!

Stable Matching

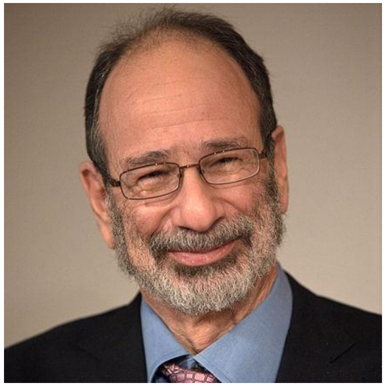
National Residency Matching Program

- National system for matching US medical school graduates to medical residencies
  - Roughly 40,000 doctors per year
  - Assignment is almost entirely algorithmic

David Gale (1921-2008)
PROFESSOR, UC BERKELEY



Lloyd Shapley
PROFESSOR EMERITUS, UCLA



Alvin Roth
PROFESSOR, STANFORD

(Centralized) Labor Markets

Markets can be asynchronous

Information is hidden

Matchings

- n doctors who need jobs $d_1, \dots, d_n$
  - a ranking of the n hospitals $h_3 > h_4 > h_1 > h_2$
- n hospitals each with one job $h_1, \dots, h_n$
  - a ranking of the n doctors $d_1 > d_4 > d_2 > d_3$

Hospital Prefs

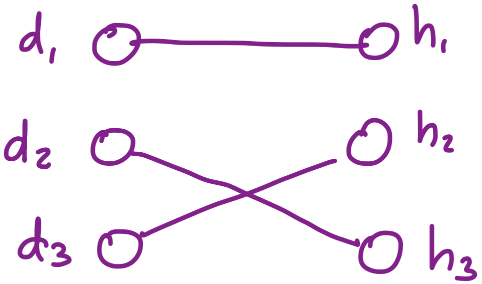
	1st	2nd	3rd	4th	5th
MGH	Bob	Alice	Dorit	Ernie	Clara
BW	Dorit	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
MTA	Alice	Dorit	Clara	Bob	Ernie
CH	Bob	Dorit	Alice	Ernie	Clara

Doctor Prefs

	1st	2nd	3rd	4th	5th
Alice	CH	MGH	BW	MTA	BID
Bob	BID	BW	MTA	MGH	CH
Clara	BW	BID	MTA	CH	MGH
Dorit	MGH	CH	MTA	BID	BW
Ernie	MTA	BW	CH	BID	MGH

Matchings

- A set of doctor hospital pairs $\{(d_1,h_1),\ (d_2,h_3),\ (d_3,h_2)\}$
  - Each doctor in at most one hosp.
  - Vize versa

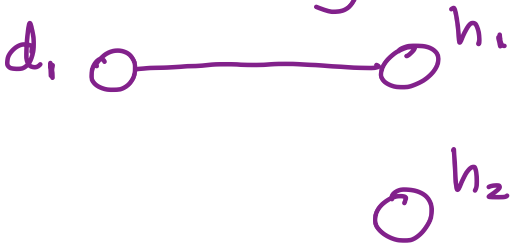


- A <u>perfect</u> matching if everyone in exactly one pair

Stable Matchings

- We want a matching with no <u>instabilities</u>

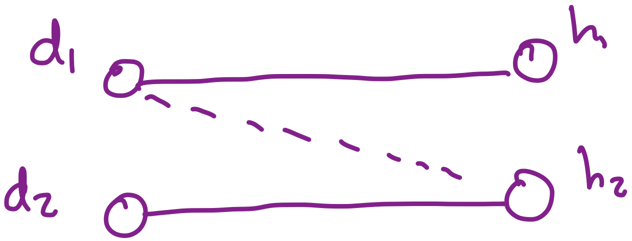
①



$h_2 \succ h_1$ by d_1

② [the reverse]

③



$d_1 : h_2 \succ h_1$

$h_2 : d_1 \succ d_2$

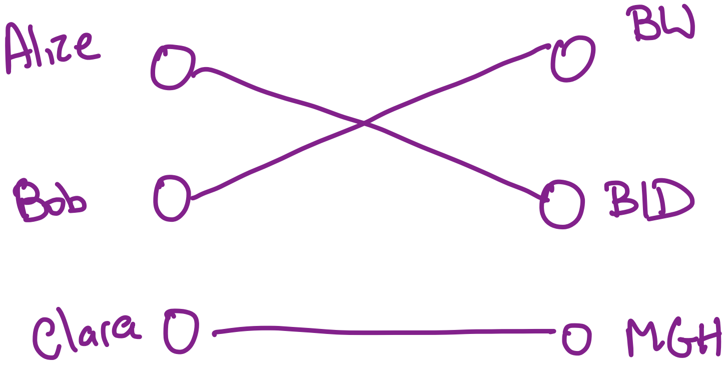
- A matching is <u>stable</u> if it has no <u>instabilities</u>

Ask the Audience

- Either find a stable matching or convince yourself that there is no stable matching

	1st	2nd	3rd
MGH	Alice	Bob	Clara
BW	Bob	Clara	Alice
BID	Alice	Clara	Bob

	1st	2nd	3rd
Alice	BW	BID	MGH
Bob	BW	MGH	BID
Clara	MGH	BID	BW



Gale-Shapley Algorithm

Assumes $\#$hosptials $\leq$ $\#$doctors

```
[Hospital offers version]
    Set $M = \emptyset$
    While there is at least one unmatched hospital
        - Pick a hospital with no doctor $h$
        - $h$ will offer a job to the highest ranked
          doctor $d$ it hasnt offered to yet
            ① $d$ has no job, so $d$ accepts (add $(d,h)$ to $M$)
            ② $d$ has a job with $h'$ and $h' > h$ to $d$
            ③ $d$ has a job with $h'$ and $h > h'$ to $d$
                (removed $(d,h')$ from $M$, add $(d,h)$ to $M$)
    Return $M$
```

One offer

Gale-Shapley Demo

	1st	2nd	3rd	4th	5th
MGH	~~Bob~~	Alice	Dorit	Ernie	Clara
BW	~~Dorit~~	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
MTA	~~Alice~~	Dorit	Clara	Bob	Ernie
CH	Bob	Dorit	Alice	Ernie	Clara

	1st	2nd	3rd	4th	5th
Alice	CH	MGH	BW	~~MTA~~	BID
Bob	BID	BW	MTA	~~MGH~~	CH
Clara	BW	BID	MTA	CH	MGH
Dorit	MGH	CH	MTA	BID	~~BW~~
Ernie	MTA	BW	CH	BID	MGH

Observations

- Any doctor who gets matched stays matched
- "Doctors move up"
- "Hospitals move down"

Gale-Shapley Algorithm: Analysis

① The algorithm always terminates and outputs a perfect matching

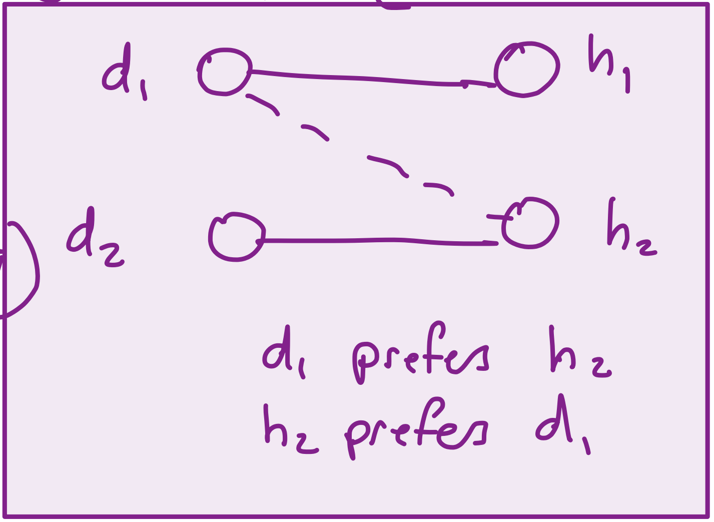
② The matching is stable ⟹ Stable matchings always exist

Gale-Shapley Algorithm: Analysis

Thm: Gale Shapley returns a stable pefect matching.

Proof Sketch: Suppose M were unstable

then we have at least one



d_1 prefers h_2
h_2 prefers d_1

- h_2 offered to d_1 first ("hospitals go down")

↳ d_1 rejected because it was paired to a better hospital than h_2

↳ d_1 accepted but later switched to h_1 ;
and it must like h_1 better since "doctors go up"

Gale-Shapley Algorithm: Analysis

Real World Impact

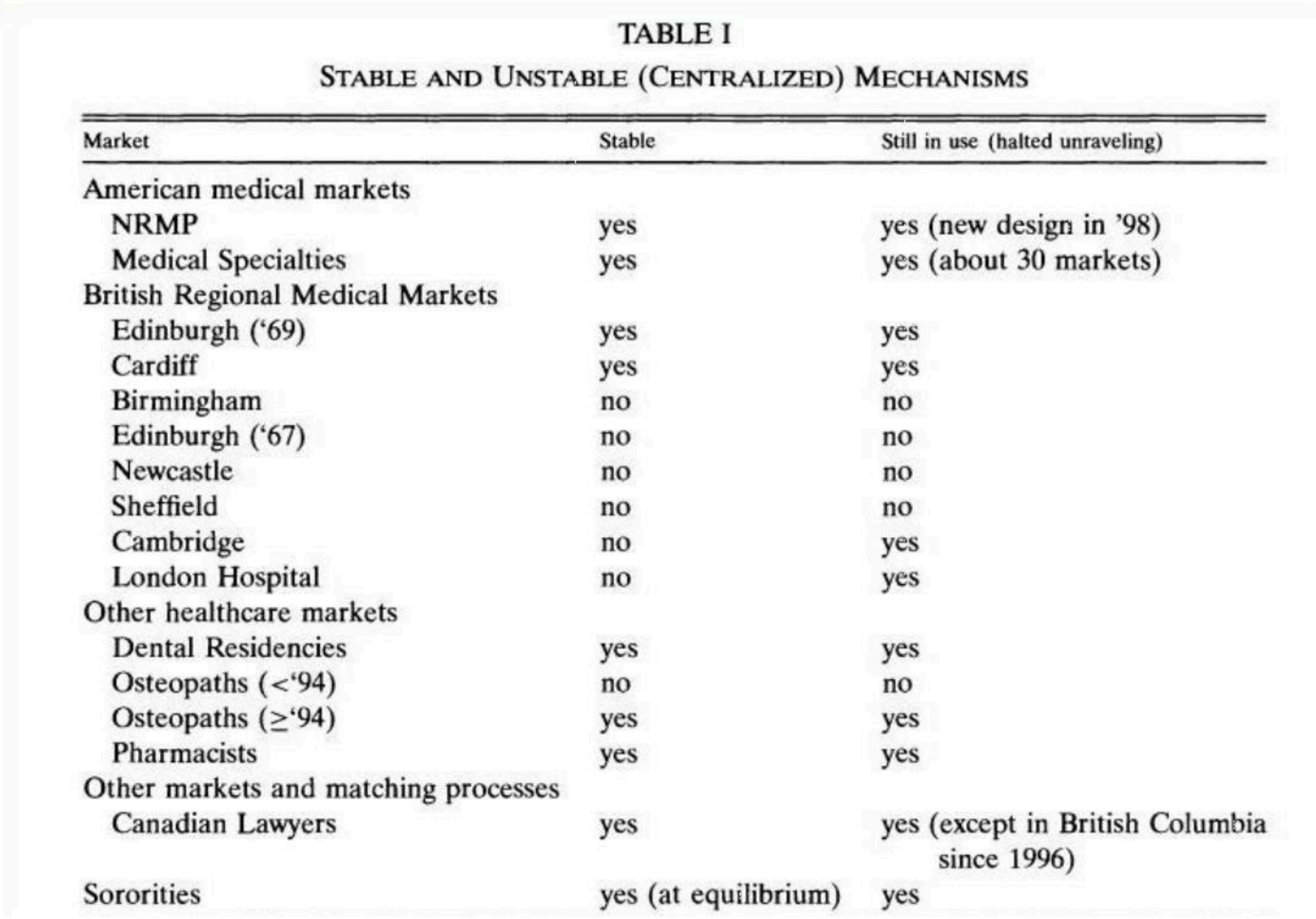


TABLE I

STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (≥'94)	yes	yes
Pharmacists	yes	yes
Other markets and matching processes		
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

Table 1. Reproduced from Roth (2002, Table 1).

Real World Challenges

- Doctors ↔ Hospitals
  - Have to deal with two-body problems
  - Have to make sure doctors do not game the system
- Kidneys ↔ Patients
  - Not all matches are feasible (blood types, immunity)
  - Certain pairs must be matched
- Students ↔ Public Schools
  - Siblings, walking zones, diversity
- Rabbis ↔ Synagogues
  - No idea why, just a fun example

2012 Nobel Prize to Lloyd Shapley and Al Roth!