CS4810 / CS7800: Advanced Algorithms — Fall 2022
Homework 5: Due Friday, November 18, 2022

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Collaboration and Honesty Policy Reminder. Collaboration in the form of discussion is allowed and encouraged. However, all forms of cheating are not allowed and will be penalized harshly. These include, but are not limited to, copying parts of an assignment from a classmate, finding answers to problems on the internet or from anyone not enrolled in the class, and plagiarizing from research papers or old posted solutions. A rule of thumb is that you should be able to walk away from discussing a homework problem with no notes and write your solution on your own.

- You must write up all solutions by yourself, and may not share any written solutions, even if you collaborate with others to solve the problem.
- You must identify all your collaborators. If you did not collaborate, write "no collaborators" or something to that effect. You may have a maximum of two collaborators per assignment, and collaboration is transitive (if you list a collaborator they must list you).
- Asking and answering questions in class forums (lectures, office hours, Piazza) is allowed and encouraged, and you do not need to list these interactions as collaborators.
- Seeking out alternative sources (e.g. classmates, textbooks, the internet) for general concepts you need (e.g. greedy algorithms, probability) is allowed and encouraged.

Assigned Problems

Problem 1 Consider the max-flow problem with a slight twist that for any edge e we should either have $f_e = 0$ or $f_e = c_e$. That is, edge e should either have no flow or it should be full with flow equal to its capacity. We call this *maximum flow with a twist* (MFt).

1.1 Formulate the MFT problem as a decision problem.

1.2 Prove that MFT is in NP.

1.3 Prove that MFT is NP-complete.

Problem 2 Consider a scheduling problem with n jobs and m machines. Each job i must be assigned to exactly one machine, and the time to complete the job is $p_i \in \mathbb{Z}_{\geq 0}$, regardless of which machine it is assigned to. The load of any machine j is the sum of processing times of the jobs assigned to it. In this problem the goal is to find an assignment of jobs to machines with minimum cost, where the cost of an assignment is the load of the machine with maximum load.

Design an efficient, greedy 2-approximation algorithm for this problem. Clearly describe your algorithm and prove that your algorithm, analyze its running time, and prove that it satisfies the desired approximation ratio.

Problem 3 You've mysteriously found yourself in the castle of an unfriendly giant. He will only let you live if you can help with his castle's lighting. The castle has n lamps and m switched. Each lamp i is connected to a nonempty subset of switches S_i. Each lamp i will turn on if an *even* number of the switches in S_i are turned on. To have a less spooky castle, the giant wants to ensure that at least half the lamps are on.

Prove that there exists a subset of switches T that you can turn on to ensure that at least half the lamps light up.

3.1 Suppose you flip each light switch independently with probability p. What is the expected number of lights that will turn on in the castle?

3.2 Design an efficient *randomized* algorithm for deciding which switches to flip such that

$$\Pr(\text{at least } n/2 \text{ lamps are turned on}) > 0$$

and prove that it satisfies this property.