CS 7800: Advanced Algorithms

Lecture 1: Intodoction + Stable Matching

Jonathan Ullman
09-09-2022

Me

- **Jonathan Ullman**
  - Call me Jon
  - Research: privacy for ML/stats

  - Office: 623 ISEC
  - Office Hours: T 3:15-5:15

The TA Team

- **Konstantina Bairaktari**
  - Office Hours: Th 3:00-5:00
  - Location: TBD

- **Rose Silver**
  - Office Hours: F 3:30-5:30
  - Location: TBD

Algorithms

What is an algorithm?

An explicit, precise, unambiguous, mechanically-executable sequence of elementary instructions for solving a computational problem.

-Jeff Erickson

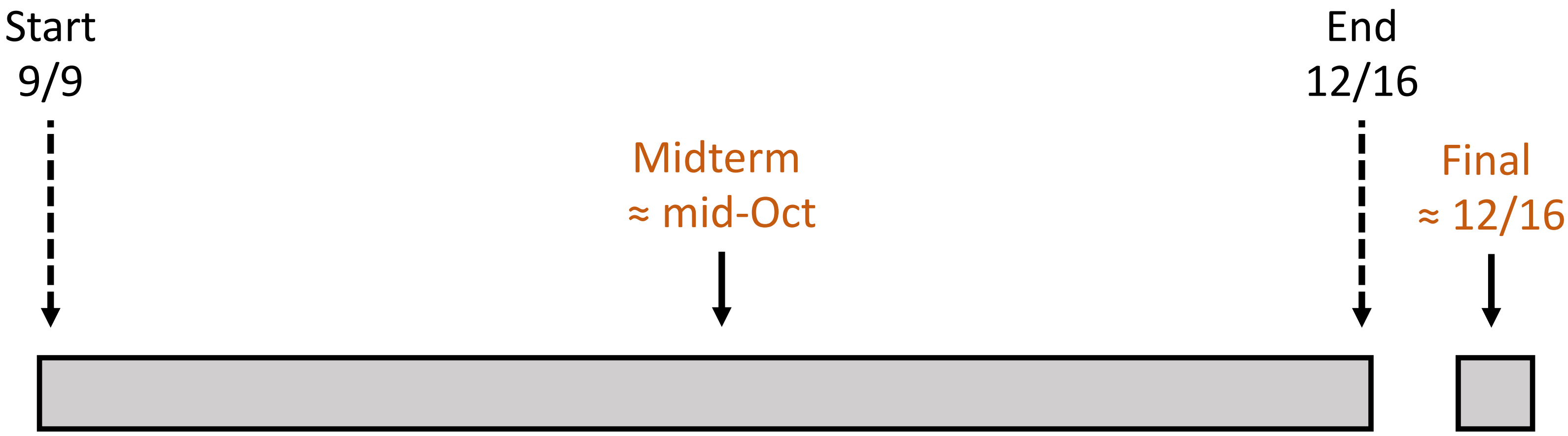
Algorithms

What is CS 7800: Advanced Algorithms?

The study of how to reason about and talk about computational problems.

- Formalize computational problems
- Learn about algorithmic techniques
- Rigorously prove properties of algorithms
- Rigorously compare algorithms
- Communicate precisely about algorithms

Course Structure

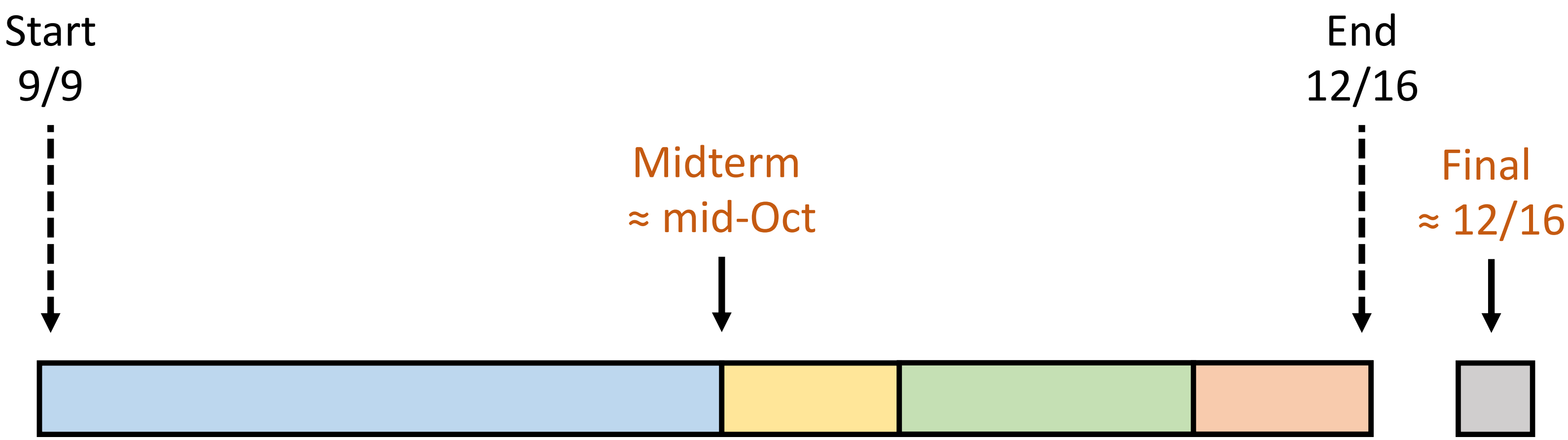


- HW x6 = 50%
- Midterm = 20%
- Final = 30%

Grades on a curve

- A/A- ≈ 50%
- B+/B ≈ 50%
- Rarely B- and lower

Course Structure



Optimization
- Greedy
- Dynamic programming
- Network flow
- Linear programming
- Reductions

Intractability

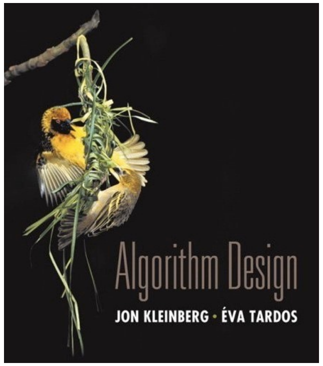
Randomized

Misc

Prereq:
- Asymptotic notation
- Proof by induction
- Graphs
- "Basic algorithms"
- Probability

Textbook

- Algorithm Design by Kleinberg and Tardos
  - We'll follow this closely in the 1^{st} half
  - Can easily find copies



- Algorithms by Jeff Erickson
  - Useful for review, alternate perspective, and some advanced topics
  - Will use this sometimes in the 2^{nd} half
  - Free on the web

Homework

- 6 HW Assignments (approximately)
  - Every two weeks (approximately)
  - Due Fridays 11:59pm
  - Late days: total of 6, but max of 2 per assignment
  - Other extensions granted rarely

- Algorithms and math questions, no programming

- **Review HW1 out now, due Friday 9/16!**

Homework Policies

- **I strongly encourage you to work together**
  - You can collaborate with up to two people per HW
  - You must list all collaborators on your solutions
  - You must write all solutions by yourself

- Rules of thumb:
  - Should be able to leave meetings without any notes
  - You should be able to explain anything you submit

Homework Policies

- Homework must be typeset in LaTeX!
  - Many good resources available
  - Many good editors available (Overleaf, TexStudio)
  - I will provide source to get you started

The Not So Short Introduction to LaTeX 2_ε

Or LaTeX 2_ε in 157 minutes

by Tobias Oetiker

Hubert Partl, Irene Hyna and Elisabeth Schlegl

Version 5.06, June 20, 2016

Homework Policies

- Homework will be submitted on Gradescope!
  - Entry code: K33NED

Discussion Forum

- We will use Piazza for discussions
  - Ask questions and help your classmates
  - Please use private messages sparingly!
  - piazza.com/northeastern/fall2022/cs7800

Course Website

http://jonathan-ullman.github.io/cs7800-f22

Home | Course Info | **Schedule**

CS 7800: Advanced Algorithms
Fall 2022

Course Schedule

Reading Code: KT = Kleinberg Tardos

Date	Topic	Reading	Notes
Fri 9/9/22	**Lecture 1: Introduction** • Course Welcome • Stable Matching [slides]	—	HW1 Out [pdf] [tex]
Tue 9/13/22	**Lecture 2: Greedy Algorithms I** • Interval Scheduling • Minimizing Lateness [slides before] [slides after]	KT 4.1-4.2	
Fri 9/16/22	**Lecture 3: Greedy Algorithms II** [slides before] [slides after]		**HW1 Due** HW2 Out
Tue 9/20/22			
Fri 9/23/22			
Tue 9/27/22			
Fri 9/30/22			

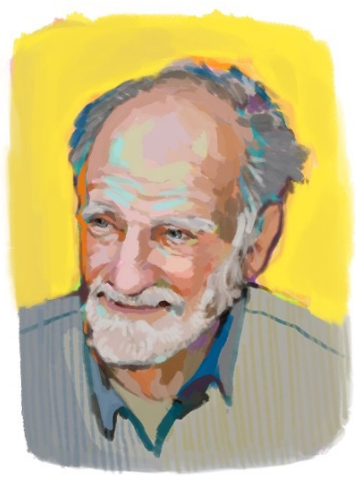
Introductions?

Stable Matching

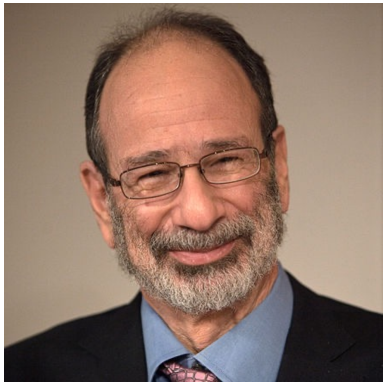
National Residency Matching Program

- National system for matching US medical school graduates to medical residencies
  - Roughly 40,000 doctors per year
  - Assignment is almost entirely algorithmic

David Gale (1921-2008)
PROFESSOR, UC BERKELEY



Lloyd Shapley
PROFESSOR EMERITUS, UCLA



Alvin Roth
PROFESSOR, STANFORD

(Centralized) Labor Markets

Matchings

n doctors   n hospitals / jobs

$\{ d_i : h_7 > h_{13} > h_2 \dots > h_{84} \} \; i \in [n]$

$\{ h_i : d_{162} > d_{112} > d_4 > \dots > d_9 \} \; i \in [n]$

Ent: MGH > BID

	1st	2nd	3rd	4th	5th
MGH	Bob	Alice	Dorit	Ernie	Clara
BW	Dorit	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
MTA	Alice	Dorit	Clara	Bob	Ernie
CH	Bob	Dorit	Alice	Ernie	Clara

	1st	2nd	3rd	4th	5th
Alice	CH	MGH	BW	MTA	BID
Bob	BID	BW	MTA	MGH	CH
Clara	BW	BID	MTA	CH	MGH
Dorit	MGH	CH	MTA	BID	BW
Ernie	MTA	BW	CH	BID	MGH

Matchings

$\mu = \{ (d_6, h_{37}), (d_8, h_2), \ldots \}$

- "Matching" no doctor/hospital appears more than once

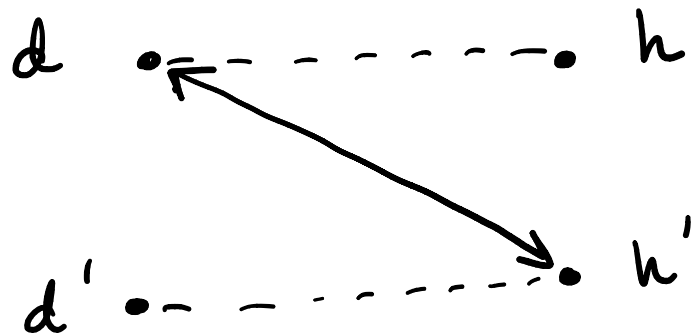
- "Perfect matching" every doctor gets a job

Stable Matchings

(d, d', h, h')

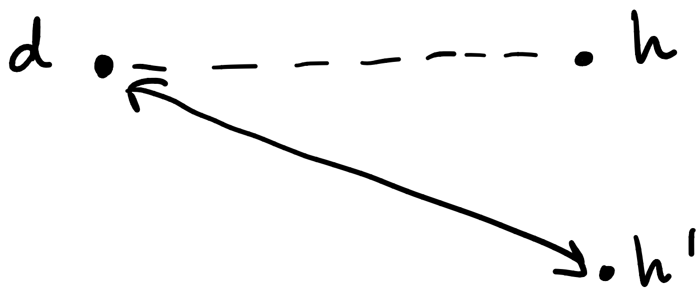
"Instability"

$d: h' > h$



$h': d > d'$

$d: h' > h$



A matching μ is stable if there are no instabilities

Ask the Audience

- Either find a stable matching or convince yourself that there is no stable matching

	1st	2nd	3rd
MGH	Alice	Bob	Clara
BW	Bob	Clara	Alice
BID	Alice	Clara	Bob

	1st	2nd	3rd
Alice	BW	BID	MGH
Bob	BW	MGH	BID
Clara	MGH	BID	BW

$\mu = \{(\text{Clara}, \text{MGH}), (\text{BW}, \text{Bob}), (\text{BID}, \text{Alice})\}$

Gale-Shapley Algorithm

Input: preferences for n doctors, n hospitals

$\mu = \emptyset$

While (there is an unmatched hospital h):

- h "offers" to their favorite d they haven't offered to yet

(1) d has no job → add (h, d) to μ

(2) d has a job at h' and h' > h → do nothing

(3) d has a job at h' but h > h' → remove (h', d), add (h, d)

Output μ

Gale-Shapley Demo

	1st	2nd	3rd	4th	5th
MGH	Bob	Alice	Dorit	Ernie	Clara
BW	Dorit	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
MTA	Alice	Dorit	Clara	Bob	Ernie
CH	Bob	Dorit	Alice	Ernie	Clara

	1st	2nd	3rd	4th	5th
Alice	CH	MGH	BW	MTA	BID
Bob	BID	BW	MTA	MGH	CH
Clara	BW	BID	MTA	CH	MGH
Dorit	MGH	CH	MTA	BID	BW
Ernie	MTA	BW	CH	BID	MGH

~~Ask the Audience~~ Observations

- Either find a stable matching or convince yourself that there is no stable matching

	1st	2nd	3rd
MGH	~~Alice~~	~~Bob~~	Clara
BW	Bob	Clara	Alice
BID	Alice	Clara	Bob

	1st	2nd	3rd
Alice	BW	BID	~~MGH~~
Bob	BW	~~MGH~~	BID
Clara	MGH	BID	BW

① Has to terminate

② Any doctor that gets an offer will always hold an offer

③ "Hospitals go down"

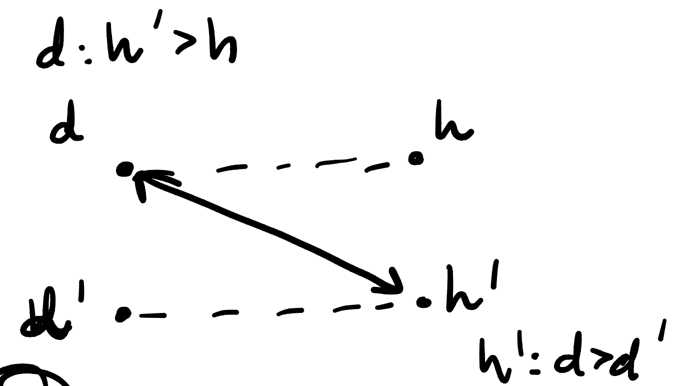
④ "Doctors go up"

Observations

Gale-Shapley Algorithm: Analysis

Why is μ a stable matching?

Suppose there were an instability



<u>Cases:</u> ① h' never offered to d (X)

② h' made an offer to d

(2a) d accepted, but later accepted h (X)

(2b) d rejected

d: h'' > h' ... h ...

at some point d has a match w/ h'' (X)

Real World Impact

TABLE I

STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (≥'94)	yes	yes
Pharmacists	yes	yes
Other markets and matching processes		
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

Table 1. Reproduced from Roth (2002, Table 1).

Real World Challenges

- Doctors ⟷ Hospitals
  - Have to deal with two-body problems
  - Have to make sure doctors do not game the system
- Kidneys ⟷ Patients
  - Not all matches are feasible (blood types, immunity)
  - Certain pairs must be matched
- Students ⟷ Public Schools
  - Siblings, walking zones, diversity
- Reform Rabbis ⟷ Synagogues
  - No idea, just a fun example

2012 Nobel Prize to Lloyd Shapley and Al Roth!